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Comparing Antenna Conical Scan Algorithms for Spacecraft Position Estimation

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I. Introduction

IN THIS Note, we will examine nonlinear estimation techniques to solve nonlinear problems that have been traditionally solved by linear methods. In the area of nonlinear estimation, a class of sampling algorithms known as Markov chain Monte Carlo (MCMC) was extensively used to obtain a solution that is often a general, non-Gaussian, nonunimodal probability distribution.

Therefore, there is a natural question to ask regarding problems that have been solved in practice by linear/Gaussian approximations: Can we do better with nonlinear methods such as MCMC? If so, how much better?

This Note examines the problem of estimating spacecraft position using scanning techniques for NASA's Deep Space Network antennas and compares different algorithms through numerical studies. As described in [1–3], the NASA Deep Space Network antennas have spacecraft trajectory programmed into them to form the antenna command. To compensate for disturbances and determine the true position of the spacecraft, circular movements are added to the antenna command trajectory in a technique known as conical scanning (ConScan). From the sinusoidal variations in the power of the signal received from the spacecraft by the antenna, the true spacecraft position can then be estimated. A least-squares method was reported in literature and used in practice for the batch processing mode. We compare this method with two other possible methods: the general linear method, which uses prior distribution of spacecraft position, and the MCMC method, which tries to solve the nonlinear problem directly by representing the desired distribution of spacecraft position with samples.

Simulations show that for the amount of data collected for ConScan batch processing in practice, all three algorithms perform essentially the same. When we artificially reduce the amount of available data, performance improvement manifests itself but the amount is dependent upon the noise level. For a low level of noise, general linear is significantly better than least squares, whereas MCMC is marginally better than general linear. For a high level of noise, general linear is marginally better than least squares, whereas MCMC is significantly better than general linear.

II. ConScan Batch Processing

Because ConScan is much faster than disturbances such as thermal deformations that cause the antenna to not point precisely toward the spacecraft, the estimation problem can be simplified and mathematically described as follows. In Fig. 1, the origin of the coordinate represents the location to which the antenna would point normally (when not performing ConScan), and a_i is its position at sampling instant i during a ConScan period. The spacecraft position s is unknown and assumed constant in the chosen coordinate system. Power measurement p_i is taken at each antenna position a_i $(i = 1, 2, \ldots, N)$ and is a nonlinear function of the norm of the pointing error $e_i \triangleq a_i - s$:

$$p_i = \gamma(||e_i||) \tag{1}$$

The nonlinear function $\gamma(\cdot)$ can be approximated at several levels. In this Note, as in [1], we consider only the following model of power measurement:

$$\gamma(||e_i||) \approx p_0\left(1 - \frac{\mu}{h^2}||e_i||^2\right) \tag{2}$$

where p_0, μ, and h are known constants.

To account for noises and disturbances that affect antenna position and power reading, we consider

$$p_i = \gamma(||a_i + w_i - s||) + v_i \tag{3}$$

where w_i and v_i are additive Gaussian noises of appropriate dimensions. The problem is thus to determine the spacecraft position s with nominal antenna positions a_i and noisy power measurements p_i. In this Note, only the batch-mode processing is considered; that is, data collected from a full ConScan period are used all at once to compute the true spacecraft position s.

III. Linear Problem

We will start by assuming that the antenna position is not perturbed (i.e., $w_i = 0$) and only the receiver power measurement is corrupted by noise (i.e., $v_i \neq 0$). Thus, the equation we are considering is

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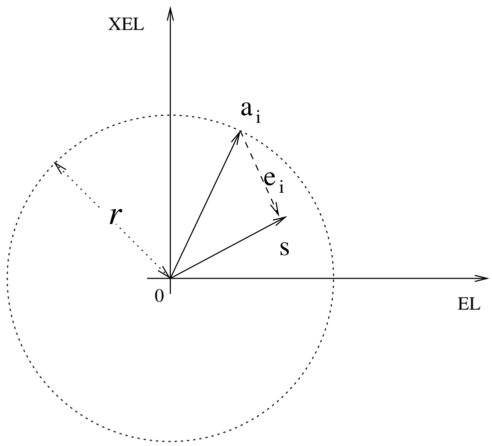

Fig. 1 Illustration of the ConScan problem setup. The origin is the nominal spacecraft position and s is the true (but unknown) position. At time instant i during a ConScan period, the antenna's position is a_i and the radius of the circle is $||a_i|| = r$. The antenna power measurement is a function of the pointing error norm $||e_i||$.

$$p_i = p_0\left(1 - \frac{\mu}{h^2}||a_i - s||^2\right) + v_i \tag{4}$$

where v_i is assumed to be independent and identically distributed normal with mean 0 and variance σ^2.

The purpose of choosing this setting is twofold: First, the connection with [1] is easier to make under this setting. Second, the solution given here serves to illustrate the linearized solution for the nonlinear problem considered in the next section.

For batch processing, we define $p \triangleq [p_1, p_2, \ldots, p_n]^T$ for an entire ConScan period with n samples. Assume that the spacecraft position s has a prior distribution that is normal with mean 0 and covariance P, denoted by

$$s \sim \mathcal{N}(0, P_s) \tag{5}$$

Our objective is to obtain the posterior distribution of s given noisy measurement p and to define a suitable estimate of s based on this posterior distribution (e.g., the posterior mean).

Now we introduce the following assumption: The nominal antenna positions a_i are symmetric on the circle in Fig. 1, that is,

$$\sum_{i=1}^{n} a_i = 0 \tag{6}$$

The preceding condition holds true in the simulations reported in [1], but was not stated as a requirement there. Our reason for this assumption will be clear in the following derivation.

From Eq. (4), it follows that

$$p_i = p_0\left(1 - \frac{\mu}{h^2}(r^2 + s^T s)\right) + \frac{2p_0\mu}{h^2}a_i^T s + v_i \tag{7}$$

where r is the radius of the circle in Fig. 1 and, consequently, $a_i^T a_i = r^2$, $i = 1, 2, \ldots, n$.

Define

$$\bar{p} \triangleq \frac{1}{n}\sum_{i=1}^{n} p_i = p_0\left(1 - \frac{\mu}{h^2}(r^2 + s^2)\right) + \frac{1}{n}\sum_{i=1}^{n} v_i \tag{8}$$

where the assumption (6) is used.

Define

$$z_i \triangleq p_i - \bar{p} = \frac{2p_0\mu}{h^2}a_i^T s + \xi_i \tag{9}$$

where

$$\xi_i \triangleq v_i - \frac{1}{n}\sum_{i=1}^{n} v_i$$

Then we can write in a compact form

$$z = As + \xi \tag{10}$$

where $z \triangleq [z_1, z_2, \ldots, z_n]^T$, $A \triangleq (2p_0\mu/h^2)[a_1, a_2, \ldots, a_n]^T$, and $\xi \triangleq [\xi_1, \xi_2, \ldots, \xi_n]^T$. Thus, the problem was reduced to a linear one, that is, estimating s from its noisy measurement z given by the linear equation (10). This is an *exact* formulation for the measurement model (4), with a symmetry assumption (6) that can easily be satisfied in practice.

It is well known (e.g., [4]) that, given the prior distribution (5) and Eq. (10), the posterior distribution of s is Gaussian [i.e., $s \sim \mathcal{N}(m_{s|z}, P_{s|z})$], where the posterior mean is given by

$$m_{s|z} = P_s A^T (A P_s A^T + R)^{-1} z = (P_s^{-1} + A^T R^{-1} A)^{-1} A^T R^{-1} z \tag{11}$$

and the posterior covariance is given by $P_{s|z} = (P_s^{-1} + A^T R^{-1} A)^{-1}$, and R is the covariance matrix of ξ (that can be calculated from the covariance of v_i).

A special case of the preceding solution is as follows: When n is large enough, we have $R \approx \sigma^2 I$, where σ is the variance of power measurement noise v_i and I is the identity matrix. If we do not assume any prior knowledge of s, we can set $P_s = \infty$. Thus, the solution (11) is reduced to

$$m_{s|z} = (A^T A)^{-1} A^T z \tag{12}$$

This is the least-squares solution given in [1] .

IV. Nonlinear Problem

In this section, we consider the case when the antenna position is also disturbed (i.e., $w_i \neq 0$). Because of the nonlinearity of the power measurement model, it will become clear from later derivations that the problem seems a truly nonlinear one. For ease of presentation, we will drop the term v_i, because the treatment is similar with its presence, and consider only the equation $p_i = \gamma(||a_i + w_i - s||)$. Using measurement model (2), it follows that

$$p_i = p_0\left(1 - \frac{\mu}{h^2}(r^2 + s^T s)\right) + \frac{p_0\mu}{h^2}\Big(2a_i^T s + 2w_i^T s - 2a_i^T w_i - w_i^T w_i\Big) \tag{13}$$

Define

$$\bar{w} \triangleq \frac{1}{n}\sum_{i=1}^{n} w_i, \qquad \xi_i \triangleq 2a_i^T w_i + w_i^T w_i, \qquad \bar{\xi} \triangleq \frac{1}{n}\sum_{i=1}^{n} \xi_i$$

Let

$$\bar{p} \triangleq \frac{1}{n}\sum_{i=1}^{n} p_i = p_0\left(1 - \frac{\mu}{h^2}(r^2 + s^T s)\right) + \frac{p_0\mu}{h^2}(2\bar{w}^T s - \bar{\xi})$$

where the assumption of symmetry (6) is used. Define

$$z_i \triangleq p_i - \bar{p} = \frac{2p_0\mu}{h^2}a_i^T s + \frac{2p_0\mu}{h^2}(w_i^T - \bar{w}^T)s + \frac{p_0\mu}{h^2}(\bar{\xi} - \xi_i) \tag{14}$$

It can be seen that estimating s from the preceding equation is a nonlinear non-Gaussian problem, because the noise w_i becomes multiplicative, and the noise term ξ_i is no longer Gaussian.

A simple approximate solution can be obtained by linearization.

A. Approximate Solution by Linearization

Because the mean of s and w_i are both zero, the linearized equation for Eq. (14) is given by

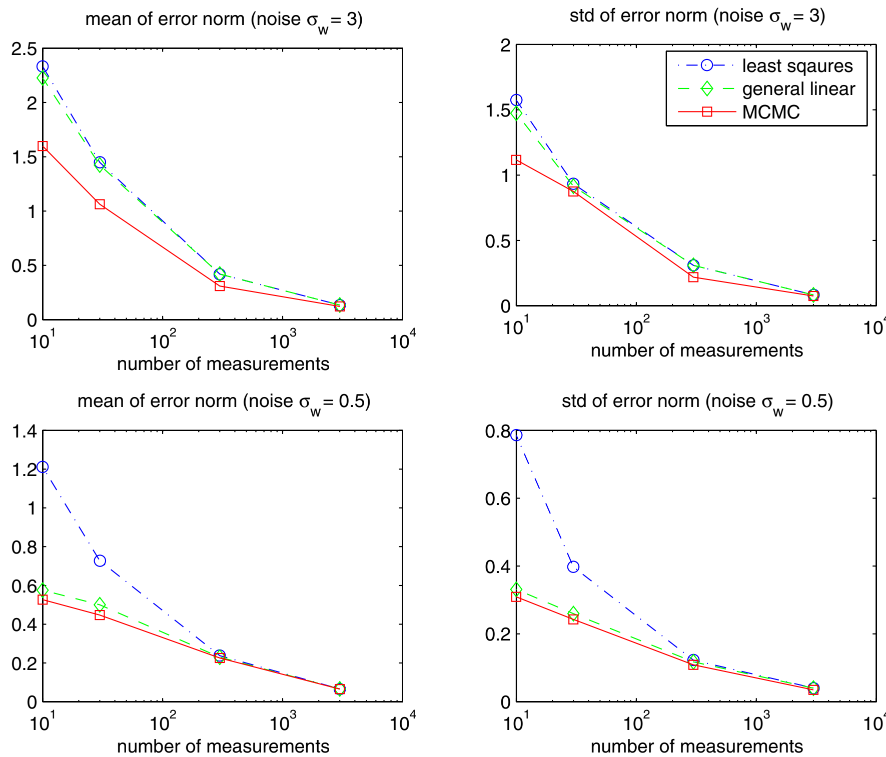

Fig. 2 Simulation results; top: high level of noise ($\sigma_w = 3$) and bottom: low level of noise ($\sigma_w = 0.5$).

$$z_i = \frac{2p_0\mu}{h^2} a_i^T s + \frac{2p_0\mu}{h^2}\left(\frac{1}{n}\sum_{i=1}^{n} a_i^T w_i - a_i^T w_i\right) \tag{15}$$

If we compare the preceding with Eq. (9), it is easy to see that they are essentially the same, and therefore the solution can be obtained in a similar fashion, which is omitted here.

Comment: We believe that a special case of the preceding solution (i.e., the least-squares solution discussed earlier) was given in [1] to also deal with antenna position disturbance.

B. Solution by Markov Chain Monte Carlo

Rather than obtaining a Gaussian approximation through linearization, we can try to obtain the posterior distribution of the spacecraft position s directly, by drawing appropriate samples from it. For easy reference, we first review how samples are used when calculating statistics by Monte Carlo integration.

If we can draw independent samples x_i $(i = 1, 2, \ldots, N)$ of a random variable x that has a probability density function (PDF) $f(x)$, then some statistics of interest [i.e., mean of $\phi(x)$] can be estimated using these samples:

$$E[\phi(x)] \triangleq \int \phi(x) f(x) dx \approx \frac{1}{N}\sum_{i=1}^{N} \phi(x_i)$$

It can be readily shown that such an estimate is unbiased and converges to the true answer as N goes to infinity.

There are efficient algorithms to draw samples from distributions such as uniform or Gaussian. However, sampling from a general PDF $f(\cdot)$ may be very difficult; this can be due to high dimensionality and/or the fact that very often we can only calculate $f(\cdot)$ up to a normalizing constant. In our ConScan problem, if we denote the PDF associated with the prior distribution of s by $f_s(\cdot)$ and denote the PDF associated with the likelihood of observing z for a given s by $f_{z|s}(\cdot|s)$, then the PDF of the desired posterior is, by Bayes's rule,

$$f(s|z) = \frac{f_{z|s}(z|s) f_s(s)}{\text{normalizing constant}} \tag{16}$$

To draw samples from this distribution, we have to resort to more sophisticated sampling techniques. MCMC is a class of sampling methods that collect samples from a Markov chain for which the stationary distribution is the desired distribution. Intuitively, after we define an appropriate Markov chain transition, we start a chain and let a sampling point "jump around" in the space of interest. At the end of a period of time called *burn in*, it will have explored the space sufficiently and "settled down" into regions of high probability. During the next period of time, called *collection*, we will record all of the points to which the chain has transitioned and use this as a set of samples. For a formal introduction to MCMC, readers are referred to [5,6]. In these simulations, we used a particular MCMC algorithm called the Metropolis–Hastings (MH) algorithm. Details about the MH algorithm are omitted due to space limitations and are available upon request.

V. Simulations

We conducted several groups of simulations for the nonlinear problem discussed in the previous section. These simulations compare three estimation methods (the least-squares solution, the general-linear solution, and the Metropolis–Hastings solution) for two cases when the noise level is high ($\sigma_w = 3$) and when the noise level is low ($\sigma_w = 0.5$), respectively. For each case and each method, simulations were carried out with various numbers of measurement points in a ConScan period (3000, 300, 30, and 10, respectively) and statistics of the results were calculated.

We observe the following from Fig. 2:

1) With 3000 measurements in a ConScan period, all methods perform essentially the same. MCMC is very computationally intensive for this case; it takes days for the simulations (run as R Language scripts[§]) to finish.

2) When we artificially reduce the number of measurements to 300, 30, and 10, we begin to see adifference in accuracy for the three methods. MCMC simulations in these cases take hours or minutes to finish.

[§]R software is available online at http://www.r-project.org/ [retrieved 3 April 2007].

3) For a high level of noise, general linear is only marginally better than least squares; that is, incorporating prior distribution of the spacecraft position does not contribute much. However, better performance is achievable with the heavy number crunching of the nonlinear MCMC method.

4) For a low level of noise, general linear is significantly better than least squares; that is, incorporating prior distribution of the spacecraft position *does* make a difference. The benefit of further number crunching by MCMC is only marginal in this case.

VI. Conclusions

In this Note, we examined the problem of estimating spacecraft position using a conical scan (ConScan) for NASA's Deep Space Network antennas using three different algorithms (general linear, least squares, and MCMC).

Extensive simulations were conducted to conclude that for the amount of data collected for ConScan batch processing in practice, all three algorithms perform essentially the same. When we artificially reduce the amount of available data, performance improvement manifests itself, but the amount is dependent upon the noise level. For a low level of noise, general linear is significantly better than least squares, whereas MCMC is marginally better than general linear. For a high level of noise, general linear is marginally better than least squares, whereas MCMC is significantly better than general linear.

It is not surprising that nonlinearity is more severe with increased level of noise, and a nonlinear method can yield better performance than linear methods, at a much higher computational cost. Thus, a nonlinear method can be used either online, if feasible, or offline to provide performance bounds for other simpler methods.

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